TECHNICAL STABILITY IN A STRAIGHT PIPELINE CONTAINING A FLOWING LIQUID

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Solutions have been obtained for the technical stability over finite and infinite time intervals and for the asymptotic technical stability [1-18] in the spatial motion of a pipeline having a rectilinear axis and with free or hinged attachment, which bears a liquid flowing at a set pressure [19-22]. Throughout its length, the pipeline has a variable cross section [21]. The liquid is taken as ideal. A nonlinear boundary-value treatment is involved, which involves three partial differential equations [23-25] with variable coefficients, which are derived from applying the theory of planar sections for long cylindrical shells (hollow rods) on the assumption that the transverse motions have little effect on the longitudinal ones. Sufficient conditions are given for the technical stability in terms of a given vector measure, which are expressed in terms of parameters containing the given pressure and speed in the liquid at the inlet. The dynamic behavior over an arbitrarily long but finite time interval is dependent on a small parameter, which in turn is dependent on the major system parameters including the internal pressure in the liquid and the speed at the inlet. Conditions are defined under which there is stability loss for a fixed pressure within the liquid. The corresponding critical inlet velocity is expressed in terms of the basic parameters. Lyapunov's direct method [20, 25-27] is applied with a comparison method [6-18, 28].

Pipeline stability is important and is dealt with in many papers [19-22, 27]. The problem is extremely complicated, so simplifying assumptions are usually made, and the stability is usually examined by means of various direct integration methods. However, there has been no study on the technical stability. The sufficient stability conditions in the Lyapunov sense for such systems have been derived [20, 27]. The present results differ substantially from the stability features in [20, 27] not only in that the conditions for technical stability are examined in a three-dimensional nonlinear formulation and for any finite present time interval but also in that the constraints on the initial stages are independent of the majoration conditions for the subsequent states during the given time interval. This approach can be used to research technical stability in more complicated pipeline problems without resort to simplifying assumptions, e.g., for curves pipelines with various forms of boundary attachment, or when there are parametric loads or turbulence, etc. It is not essential to meet the conditions for negative definiteness in the total derivative of the Lyapunov function by virtue of the initial boundary-value treatment in this approach, in contrast to stability in the Lyapunov sense, which extends the parameter range that can be used. The comparison method provides a fuller allowance for the factors that influence the stability.

1. Nonlinear Boundary-Value Treatment for the Motion of a Pipeline Containing a Flowing Liquid. We consider a long flexible pipe containing a flowing ideal liquid under the conditions stated below. We assume that the pipeline constitutes a homogeneous isotropic body and as a shell is deformed geometrically nonlinearly [24]. The symbols are: $m_1(s)$ the mass of unit length, which is dependent on the scalar coordinate s of the point on the axis line; $m_2(s)$ the mass of liquid per unit length; ρ_1 and ρ_2 the densities of the tube material and liquid; $F_1(s)$, $F_2(s)$ the area of any cross section of the tube and the area of the hole in it; ℓ tube length; h average thickness; P pressure in liquid; and $\mathbf{f}$ a vector characterizing the distribution of the interaction between the pipeline and the liquid, which for a tube whose area in the lumen varies with the coordinate s is [21] $\mathbf{f} = -P(\partial F_2(s)/\partial s)\mathbf{e}_1 + \tilde{f}_2\mathbf{e}_2 + \tilde{f}_3\mathbf{e}_3$; with $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$ unit vectors for the current time-dependent configuration under small deformations, with $\mathbf{e}_2$, and $\mathbf{e}_3$ directed along the principal axes of the cross section; and $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ vectors for the relative velocity of the liquid, where $w_0F_{20} = mF_2(s)$, $m_{20}w_0 = m_2w$; for small deformations and finite angles of rotation, the basic vectors γ_1, γ_2 and $\mathbf{q}$ for a current configuration are taken as mutually orthogonal and directed along the principal axes

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of inertia in the pipeline for any point on the axial line; $\mathbf{q}$ the vector for the external distributed forces on the tube; and γ_1 and γ_2 are the vectors for the distributed forces caused by the force fields correspondingly for the tube and liquid; $\mathbf{Q}^{(1)}$ the internal-force vector; $\mathbf{Q}^{(1)} = Q_1^{(1)}\mathbf{e}_1 + Q_2^{(1)}\mathbf{e}_2 + Q_3^{(1)}\mathbf{e}_3$; with $Q_1^{(1)}$ the axial force and $Q_2^{(1)}, Q_3^{(1)}$ the shearing forces; while $\mathbf{M}$ is the vector for the internal moments: $\mathbf{M} = M_1\mathbf{e}_1 + M_2\mathbf{e}_2 + M_3\mathbf{e}_3$; with M_1 the torque and M_2 and M_3 the bending moments; $\mathbf{v}$ the vector for the absolute velocity of the center of gravity of an element ds in the pipeline; ν Poisson's ratio; E Young's modulus; and $\mathbf{e}_{10}$, $\mathbf{e}_{20}$, and $\mathbf{e}_{30}$ an orthogonal system of unit vectors for the unperturbed equilibrium state, with $\mathbf{e}_{10}$ along the axial line in the sense of the liquid motion, while $\mathbf{e}_{i0}$ (i = 1, 2, 3) are independent of s, which corresponds to considering the unperturbed pipeline with rectilinear axis; and $\boldsymbol{\mu} = (\mu_1, \mu_2, \mu_3)$ the distributed moment acting on the pipeline, which is stationary with respect to s, and we recall that the flow of an ideal incompressible liquid does not produce moments acting on the pipeline.

We write the nonlinear scalar equations for the motion of the pipeline containing the liquid in terms [21] of the corresponding vector equation for the forces and that for the equilibrium for the moments in the system:

$$\left[m_1(s) + m_2(s)\right]\frac{\partial \mathbf{v}}{\partial t} + m_2(s)\left(\frac{\partial \mathbf{v}}{\partial s} w + \frac{\partial \mathbf{w}}{\partial t}\right) = \frac{\partial \mathbf{Q}}{\partial s} + \mathbf{q} + \boldsymbol{\gamma},$$
$$\mathbf{Q} = \mathbf{Q}^{(1)} - [PF_2(s) + m_2(s)w^2(s)]\,\mathbf{e}_1; \tag{1.1}$$

$$\boldsymbol{\gamma} = \boldsymbol{\gamma}_1 + \boldsymbol{\gamma}_2, \quad \boldsymbol{\gamma} = (\gamma^{(1)}, \gamma^{(2)}, \gamma^{(3)});$$
$$\partial \mathbf{M}/\partial s = \mathbf{Q}\times\mathbf{e}_1 + \boldsymbol{\mu}. \tag{1.2}$$

We project (1.1) and (1.2) on the unit vectors for the $\mathbf{e}_{10}$ basis for the unperturbed state with a linear approximation for the displacement vector on a standard approach [24]. As we envisage the pipeline as long, we assume that longitudinal motions will influence the bending motion substantially but not vice versa. Then after suitable transformations via the [24] relations, we get a nonlinear system of equations of motion in dimensionless terms written in terms of the displacements:

$$\frac{\partial^2 u_1}{\partial t^2} + 2\tilde{w}_1\rho\frac{\partial^2 u_1}{\partial t\,\partial s} = \frac{\partial^2 u_1}{\partial s^2}\left[1 - (\tilde{p}_1 + \tilde{w}_1^2)\right] + f_1(s)^{-1}\frac{\partial f_1(s)}{\partial s}\frac{\partial u_1}{\partial s} + \tilde{q}_1 + \tilde{\gamma}_1,$$
$$\frac{\partial^2 u_i}{\partial t^2} + 2\tilde{w}_i\rho\frac{\partial^2 u_i}{\partial t\,\partial s} = -\frac{\partial^4 u_i}{\partial s^4} - (\tilde{p}_i + \tilde{w}_i^2)\frac{\partial^2 u_i}{\partial s^2} + A_i^{(1)}\frac{\partial}{\partial s}\left(\frac{\partial u_i}{\partial s}\frac{\partial u_1}{\partial s}\right) +$$
$$A_i^{(2)}\frac{\partial f_1(s)}{\partial s}\frac{\partial u_i}{\partial s}\frac{\partial u_1}{\partial s} + \frac{\partial u_i}{\partial s}(\bar{q}_i + \bar{\gamma}_i) + \tilde{q}_i + \tilde{\gamma}_i, \quad i = 2, 3. \tag{1.3}$$

The pipeline is considered subject to boundary conditions corresponding to free mounting or hinged attachment at the ends [22, 23]:

$$u_j(t, 0) = u_j(t, 1) = 0, \; j = 1, 2, 3,$$
$$\frac{\partial^2 u_j}{\partial s^2}(t, 0) = \frac{\partial^2 u_j}{\partial s^2}(t, 1) = 0, \quad j = 1, 2, 3, \tag{1.4}$$

and the initial conditions

$$u_j(t, s)\big|_{t=t_0} = k_j(s), \quad j = 1, 2, 3,$$
$$\left.\frac{\partial u_j(t, s)}{\partial t}\right|_{t=t_0} = g_j(s), \quad j = 1, 2, 3. \tag{1.5}$$

Here $\rho = \{m_{20}f_2(s)/[m_{10}f_1(s) + m_{20}f_2(s)]\}^{1/2}$; $\tilde{p}_1 = PF_{20}f_2(s)/EF_{10}f_1(s)\delta$; $\tilde{p}_2 = PF_{20}f_2(s)l/EI_3(s)\delta$; $\tilde{p}_3 = PF_{20}f_2(s)l^2/EI_2(s)\delta$; $\tilde{q}_1 = q_1l^2/EF_{10}f_1(s)h\delta$; $\tilde{q}_2 = q_2l^4/EI_3(s)h\delta$; $\tilde{q}_3 = q_3l^4/EI_2(s)h\delta$; $\bar{q}_2 = q_1l^3/EI_3(s)\delta$; $\bar{q}_3 = q_1l^3/EI_2(s)\delta$; $\tilde{\gamma}_1 = \gamma^{(1)}l^2/EF_{10}f_1(s)h\delta$; $\tilde{\gamma}_2 = \gamma^{(2)}l^4/EI_3(s)h\delta$; $\tilde{\gamma}_3 = \gamma^{(3)}l^4/EI_2(s)h\delta$; $\bar{\gamma}_2 = \gamma^{(1)}l^3/EI_3(s)\delta$; $\bar{\gamma}_3 = \gamma^{(1)}l^3/EI_2(s)\delta$; $\tilde{w}_1 = w_0(m_{20}/Ef_1(s)f_2(s)\delta)^{1/2}$; $\tilde{w}_2 = w_0l(m_{20}/Ef_2(s)I_3(s)\delta)^{1/2}$; $\tilde{w}_3 = w_0l(m_{20}/Ef_2(s)I_2(s)\delta)^{1/2}$; $A_2^{(1)} = F_{10}f_1(s)hl/I_3(s)$; $A_2^{(2)} = F_{10}lh/I_3(s)$; $A_3^{(1)} = F_{10}f_1(s)lh/I_2(s)$; $A_3^{(2)} = F_{10}lh/I_2(s)$; $\delta = (1-\nu)[(1+\nu)(1-2\nu)]^{-1}$; with $I_2(s)$ and $I_3(s)$ the corresponding moments of inertia for the tube cross section. The areas of intersection of the tube and the lumen in it are $F_1(s) = F_{10}f_1(s)$, $F_2(s) = F_{20}f_2(s)$, and $m_i(s) = \rho_iF_i(s) = \rho_iF_{i0}f_i(s) = m_{i0}f_i(s)$, $i = 1, 2$. A zero in the subscripts relates to a quantity at the inlet to the tube. As

$f_1(s)$, $f_2(s)$, $I_2(s)$, $I_3(s)$ are functions of s, (1.3) have coefficients dependent on s, which causes additional difficulties in examining the initial process.

We assume that the given $k_j(s)$, $g_j(s)$ in (1.5), which satisfy the necessary conditions for matching at the end of the pipeline, will ensure that (1.3)-(1.5) has a unique solution in the class of continuous functions of t and s having continuous derivatives with respect to t and s of the necessary orders.

2. Sufficient Conditions for Technical Stability. The generative system [15, 16] for (1.3) is taken as a nonlinear homogeneous one (i.e., for $q_j = 0$, $\gamma^{(0)} = 0$, $j = 1, 2, 3$) corresponding to (1.3), which we consider with boundary conditions (1.4) and (1.5). The generative system has the trivial solution $u_j(t, s) \equiv 0$ ($j = 1, 2, 3$), which we take as corresponding to the unperturbed equilibrium state. We examine the technical stability in the dynamic behavior with the perturbations described by (1.3)-(1.5), for which we use Lyapunov's second method with the comparison method. We take the vector functional

$$V[u_1, u_2, u_3, t] = \{V_j[u_j, t], j = 1, 2, 3\},$$
$$V_1[u_1, t] = \int_0^t ds\left[\left(\frac{\partial u_1}{\partial s}\right)^2 - (\tilde{p}_1 + \tilde{w}_1^2)\left(\frac{\partial u_1}{\partial s}\right)^2 + \left(\frac{\partial u_1}{\partial t}\right)^2\right], \tag{2.1}$$
$$V_i[u_i, t] = \int_0^1 ds\left[\left(\frac{\partial^2 u_i}{\partial s^2}\right)^2 - (\tilde{p}_i + \tilde{w}_i^2)\left(\frac{\partial u_i}{\partial s}\right)^2 + \left(\frac{\partial u_i}{\partial t}\right)^2\right], \quad i = 2, 3.$$

We write the total derivative for (2.1) by virtue of (1.3)-(1.5) after suitable simplifications as

$$\frac{dV_1[u_1, t]}{dt} = 2\int_0^1 ds\left[\left(\frac{\partial(\tilde{p}_1 + \tilde{w}_1^2)}{\partial s} + f_1(s)^{-1}\frac{\partial f_1(s)}{\partial s}\right)\times\frac{\partial u_1}{\partial t}\frac{\partial u_1}{\partial s} + \frac{\partial u_1}{\partial t}\left(2\frac{\partial\left(\tilde{w}_1\rho\frac{\partial u_1}{\partial t}\right)}{\partial s} + \tilde{q}_1 + \tilde{\gamma}_1\right)\right],$$
$$\frac{dV_i[u_i, t]}{dt} = 2\int_0^1 ds\left\{\frac{\partial u_i}{\partial t}\left[\frac{\partial u_i}{\partial s}\left(\frac{\partial(\tilde{p}_i + \tilde{w}_i^2)}{\partial s} + A_i^{(1)}\frac{\partial^2 u_1}{\partial s^2} + A_i^{(2)}\frac{\partial f_1(s)}{\partial s}\frac{\partial u_1}{\partial s} + \right.\right.\right. \tag{2.2}$$
$$\left.\left.\left. + \bar{q}_i + \bar{\gamma}_i\right) + A_i^{(1)}\frac{\partial^2 u_i}{\partial s^2}\frac{\partial u_1}{\partial s} + 2\frac{\partial}{\partial s}\left(\tilde{w}_i\rho\frac{\partial u_i}{\partial t}\right)\right] + \frac{\partial u_i}{\partial t}(\tilde{q}_i + \tilde{\gamma}_i)\right\}, \quad i = 2, 3.$$

We consider the vector measure

$$\rho(u) = \{\rho_1(u_1), \rho_2(u_2), \rho_3(u_3)\}, \; \rho_1(u_1) = \sup_s (u_1)^2 + \int_0^1 ds\left[\left(\frac{\partial u_1}{\partial s}\right)^2 + \left(\frac{\partial u_1}{\partial t}\right)^2\right], \tag{2.3}$$
$$\rho_i(u_i) = \sup_s (u_i)^2 + \sup_s\left(\frac{\partial u_i}{\partial s}\right)^2 + \int_0^1 ds\left[\left(\frac{\partial^2 u_i}{\partial s^2}\right)^2 + \left(\frac{\partial u_i}{\partial t}\right)^2\right], \quad i = 2, 3.$$

We use the inequalities [20]

$$\int_0^1 ds\left(\frac{\partial^2 u_j}{\partial s^2}\right)^2 \geqslant \pi^2\int_0^1 ds\left(\frac{\partial u_j}{\partial s}\right)^2 \geqslant \pi^4\int_0^1 ds u_j^2, \; \int_0^1 ds\left(\frac{\partial^2 u_j}{\partial s^2}\right)^2 \geqslant \sup_s\left(\frac{\partial u_j}{\partial s}\right)^2,$$
$$\int_0^1 ds\left(\frac{\partial u_j}{\partial s}\right)^2 \geqslant \sup_s u_j^2, \quad j = 1, 2, 3,$$

to find a lower bound for $V_j[u_j, t]$ ($j = 1, 2, 3$) along the solution to (1.3)-(1.5):

$$2V_1[u_1(t, s), t] \geqslant 2\int_0^1 ds\left[\left(\frac{\partial u_1}{\partial s}\right)^2 - (\tilde{p}_1 + \tilde{w}_1^2)\left(\frac{\partial u_1}{\partial s}\right)^2\right] +$$
$$+ \int_0^1 ds\left(\frac{\partial u_1}{\partial s}\right)^2 - \int_0^1 ds(\tilde{p}_1 + \tilde{w}_1^2)\left(\frac{\partial u_1}{\partial t}\right)^2 \geqslant [1 - (p_1 + w_1^2)]\sup_s (u_1)^2 +$$
$$+ [1 - (p_1 + w_1^2)]\int_0^1 ds\left[\left(\frac{\partial u_1}{\partial s}\right)^2 + \left(\frac{\partial u_1}{\partial t}\right)^2\right] = [1 - (p_1 + w_1^2)]\,\rho(u_1(t, s)),$$

$$3V_i[u_i(t,s),t] \geqslant \pi^2 \int_0^1 ds \left(\frac{\partial u_i}{\partial s}\right)^2 - (p_i + w_i^2) \int_0^1 ds \left(\frac{\partial u_i}{\partial s}\right)^2 + 2\int_0^1 ds \left(\frac{\partial^2 u_i}{\partial s^2}\right)^2 -$$
$$- \frac{2}{\pi^2}(p_i + w_i^2) \int_0^1 ds \left(\frac{\partial^2 u_i}{\partial s^2}\right)^2 + [1-(p_i+w_i^2)] \int_0^1 ds \left(\frac{\partial u_i}{\partial t}\right)^2 \geqslant [1-(p_i+w_i^2)]\,\rho_i(u_i(t,s)),\ i=2,3, \tag{2.4}$$
$$p_j = \sup_s(\tilde{p}_j),\ w_j = \sup_s(\tilde{w}_j),\quad j=1,2,3.$$

Then

$$V_1[u_1(t,s),t] \geqslant 2^{-1}[1-(p_1+w_1^2)]\rho_1(u_1(t,s)),$$
$$V_i[u_i(t,s),t] \geqslant 3^{-1}[1-(p_i+w_i^2)]\rho_i(u_i(t,s)),\ i=2,3. \tag{2.5}$$

Then the functionals $V_j[u_j(t, s), t]$ ($j = 1, 2, 3$) are positive-definite with respect to the measure $p(u)$ if $0 \leq p_j + w_j^2 < 1$, $j = 1, 2, 3$. We examine the case

$$0 < p_j + w_j^2 < 1,\ j=1,2,3. \tag{2.6}$$

Then $\mu_j = 1 - (p_j + w_j^2)$ ($j = 1, 2, 3$) have the meaning of small positive parameters: $\mu_j \in (0, 1]$ ($j = 1, 2, 3$), with which we specify in advance an arbitrarily large but finite time interval T within which we consider the dynamic behavior of (1.3)-(1.5), namely we put $T = [t_0, L\bar{\mu}^{-1}]$, where $t_0 \geqslant 0$, and L is a specified and arbitrarily large constant, which in general is dependent on the parameters characterizing the pipeline reliability [24, 27]; we take $\bar{\mu}^{-1} = \min\{\mu_1^{-1}, \mu_2^{-1}, \mu_3^{-1}\}$. We use the monotone feature of the product for any natural quantities to get for (2.5) that

$$\frac{1}{(\mu_1+t)^2} V_1[u_1(t,s),t] \geqslant \frac{\mu_1}{2(\mu_1+t)^2}\rho_1(u_1(t,s)),$$
$$\frac{1}{(\mu_i+t)^2} V_i[u_i(t,s),t] \geqslant \frac{\mu_i}{3(\mu_i+t)^2}\rho_i(u_i(t,s)),\ i=2,3. \tag{2.7}$$

We denote the right-hand sides in (2.2) along the solution to the initial (1.3)-(1.5) system correspondingly by $N_j(t, s)$, $j = 1, 2, 3$. We use (2.2), (2.5), (2.7) to form the functions

$$\overline{\Phi}_1(t,\tilde{p}_1,\tilde{w}_1) = N_1(t,s) - \frac{\mu_1}{2(\mu_1+t)^2}\rho_1(u_1(t,s)),\ \overline{\Phi}_i(t,\tilde{p}_i,\tilde{w}_i) = N_i(t,s) - \frac{\mu_i}{3(\mu_i+t)^2}\rho_i(u_i(t,s)),\quad i=2,3. \tag{2.8}$$

The following inequalities apply:

$$\overline{\Phi}_1(t,\tilde{p}_1,\tilde{w}_1) \leqslant N_1(t,s) - \frac{\mu_1}{2(\mu_1+t)^2}\sup_s|u_1(t,s)|^2,$$
$$\overline{\Phi}_i(t,\tilde{p}_i,\tilde{w}_i) \leqslant N_i(t,s) - \frac{\mu_i}{3(\mu_i+t)^2}\sup_s|u_i(t,s)|^2,\quad i=2,3. \tag{2.9}$$

Let $\Phi_j(t, \tilde{p}_j, \tilde{w}_j)$ be known bounded continuous functions of the variable t, which are dependent on $\tilde{p}_j$, $\tilde{w}_j$ as parameters and which majorize the right sides of (2.9). In particular, along the solutions to (1.3)-(1.5) one can put

$$\Phi_j(t,\tilde{p}_j,\tilde{w}_j) = \sup_s|N_j(t,s)| - \frac{\mu_j}{\beta(\mu_j+t)^2} \times \sup_s|u_j(t,s)|,\quad j=1,\ \beta=2;\ j=2,3,\ \beta=3.$$

Along the solution to the initial (1.3)-(1.5) we get

$$\frac{dV_j[\mu_j(t,s),t]}{dt} \leqslant \frac{1}{(\mu_j+t)^2} V_j[u_j(t,s),t] + \Phi_j(t,\tilde{p}_j,\tilde{w}_j),\quad j=1,2,3. \tag{2.10}$$

Inequalities (2.10) enable one to consider a system of three linear inhomogeneous ordinary differential equations of fairly simple form [6-18, 28]:

$$\frac{dy_j}{dt} = \frac{1}{(\mu_j+t)^2} y_j + \Phi_j(t,\tilde{p}_j,\tilde{w}_j),\ j=1,2,3 \tag{2.11}$$

subject to the initial conditions

$$y_j(t_0) = y_j^0 \geqslant V_j[u_j(t_0),t_0],\ j=1,2,3. \tag{2.12}$$

The homogeneous system corresponding to (2.11) splits up into three independent scalar first-order differential equations, so each equation in (2.11) is integrated independently. Clearly, in (2.12) we have

$$V_1[u_1(t_0), t_0] = \int_0^1 ds \left\{ \left(\frac{\partial k_1(s)}{\partial s} \right)^2 [1 - (\tilde{p}_1 + \tilde{w}_1^2)] + g_1^2(s) \right\},$$
$$V_i[u_i(t_0), t_0] = \int_0^1 ds \left\{ \left(\frac{\partial^2 k_i(s)}{\partial s^2} \right)^2 - [\tilde{p}_i + \tilde{w}_i^2] \left(\frac{\partial k_i(s)}{\partial s} \right)^2 + g_i^2(s) \right\}, \quad i = 2, 3 \tag{2.13}$$

on the assumption that $k_j(s)$, $g_j(s)$ ($j = 1, 2, 3$) have the necessary regularity with respect to $s \in [0, 1]$. The features of (2.11) give us a solution continuous in the interval T:

$$y_j(t, \tilde{p}_j, \tilde{w}_j) = \exp[-1/(\mu_j + t)] \int_{t_0}^{t} d\tau \exp[1/(\mu_j + \tau)] \Phi_j(t, \tilde{p}_j, \tilde{w}_j) +$$
$$y_j^0 \exp[1/(\mu_j + t_0)] \exp[-1/(\mu_j + t)], \quad j = 1, 2, 3, \tag{2.14}$$

which is dependent on the parameters $\tilde{p}_j$, $\tilde{w}_j$. A standard theorem on differential inequalities [28] gives us for $t \in T$ along the solution to (1.3)-(1.5) a system of bounds

$$V_j[u_j(t, s), t] \leqslant y_j(t, \tilde{p}_j, \tilde{w}_j), \; j = 1, 2, 3. \tag{2.15}$$

As the $y_j(t, \tilde{p}_j, \tilde{w}_j)$ ($j = 1, 2, 3$) are continuous in T, (2.15) gives the technical stability over a finite time interval with respect to the given measure $\rho(u)$.

If the $\Phi_j(t, \tilde{p}_j, \tilde{w}_j)$ are such that $\int_0^t \exp[1/(\mu_j+\tau)] \times \Phi_j(\tau, \tilde{p}_j, \tilde{w}_j)\, d\tau$ ($j = 1, 2, 3$) are continuous bounded functions for any time interval $T \subseteq T_1 \equiv [t_0, +\infty)$ and increase in each $T \subseteq T_1$ not more rapidly than correspondingly $\exp[1/(\mu_j + t)]$, (2.14) shows that (1.3)-(1.5) is technically stable with respect to the $\rho(u)$ measure over an infinite time interval, i.e., in that case one can state preset continuous bounded functions $B_j(t)$ defined for each $T \subseteq T_1$ and such that the bounds are

$$\int_{t_0}^{t} d\tau \exp[1/(\mu_j + \tau)] \Phi_j(\tau, \tilde{p}_j, \tilde{w}_j) \leqslant B_j(t) \exp[1/(\mu_j + t)], \quad j = 1, 2, 3 \tag{2.16}$$

subject to the inequalities

$$B_j(t) + y_j^0 \exp[1/(\mu_j + t)] \exp[-1/(\mu_j + t)] \geqslant 0,$$
$$j = 1, 2, 3.$$

Also, if

$$\exp[1/(\mu_j + t)] \geqslant \int_{t_0}^{t} \exp[1/(\mu_j + \tau)] \Phi_j(\tau, \tilde{p}_j, \tilde{w}_j)\, d\tau, \quad j = 1, 2, 3 \tag{2.17}$$

for all $T \subseteq T_1$, (1.3)-(1.5) is also technically stable on the given measure over an infinite time interval T_1, since in that case for $t \to +\infty$

$$y_j(t, \tilde{p}_j, \tilde{w}_j) \to 1 + y_j^0 \exp[1/(\mu_j + t_0)],$$
$$j = 1, 2, 3. \tag{2.18}$$

If (1.3)-(1.5) is technically stable in T_1 and also

$$y_j(t, \tilde{p}_j, \tilde{w}_j) \to 0 \text{ for } t \to +\infty, \; j = 1, 2, 3, \tag{2.19}$$

then it is technically asymptotically stable on $\rho(u)$. In particular, (2.19) will be obeyed if

$$\exp\left[-1/(\mu_j + t)\right] \int_{t_0}^{t} \exp[1/(\mu_j + \tau)] \Phi_j(\tau, \tilde{p}_j, \tilde{w}_j)\, d\tau \to$$
$$-y_j^0 \exp[1/(\mu_j + t_0)], \; t \to +\infty, \; j = 1, 2, 3. \tag{2.20}$$

Then (2.16)-(2.20) together with (2.6) will represent the sufficient conditions for the corresponding technical stability.

We derive the critical inlet speed w_0^{cr} for a given pressure in the liquid. As $1 - (p_k + w_k^2) \leqslant 1 - (\tilde{p}_k + \tilde{w}_k^2)$, we solve the system

$$\tilde{p}_j + \tilde{w}_j^2 = 1,\ j = 1, 2, 3. \tag{2.21}$$

From (2.21) we get

$$w_0^{cr} = \{f_2(s)\,[3EF_{10}f_1(s)\,I_2(s)\,I_3(s)\,\delta - PF_{20}f_2(s)\,(I_2(s)\,I_3(s) + l^2F_{10}f_1(s)\,[I_2(s) + I_3(s)])]\}^{1/2}\{m_{20}\,[I_2(s)\,I_3(s) + l^2F_{10}f_1(s)\,(I_2(s) + I_3(s))]\}^{-1/2}.$$

Consequently, those sufficient conditions for technical stability on $\rho(u)$ are not met if

$$p_j + w_j^2 \geqslant 1,\ j = 1, 2, 3. \tag{2.22}$$

If with the initial conditions (2.12) and the bounds for $y_j(t, \tilde{p}_j, \tilde{w}_j)$ one has

$$V_j[u_j(t, s), t] > y_j(t, \tilde{p}_j, \tilde{w}_j)\ \ (j = 1, 2, 3) \tag{2.23}$$

for one instant t in the finite or infinite time interval, then (1.3)-(1.5) will be technically unstable in the finite or infinite time interval. In particular, (2.15) implies that one of the conditions for technical instability in that system is

$$y_j(t, \tilde{p}_j, \tilde{w}_j) \to +\infty\ \ (j = 1, 2, 3) \tag{2.24}$$

for $t \in T$ or $t \in T_1$. For example, if the initial moment in (2.14) is $t_0 = 0$, then (2.24) may apply for $\mu_j \to 0$ (j = 1, 2, 3) for any $t \in T$ or $t \in T_1$.

The effects for a given pressure in a liquid in stationary flow are similar to those of an axial compressive force.

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APPLICATION OF THE HOLOGRAPHIC INTERFEROMETRY METHOD TO DETERMINE THE STRESS INTENSITY FACTOR

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Methods are analyzed for the determination of the stress intensity factor K_I by means of experimentally found displacements in the area of a crack apex. The method of holographic interferometry for recording holograms by the scheme of opposing beams is used to measure the displacements. In order to raise the hologram quality and the accuracy it is recommended to superpose a high-frequency metallized raster on the structure surface. A method is described for finding K_I by opening the crack. Examples are presented of investigation of a calibration specimen and a ribbed panel with a fatigue crack.

When studying structures with cracks the stress intensity factor can be found from experimentally measured displacements in the area of the crack apex [1-3]. Both the displacements u, v in the plane of the specimen [1, 2] and the displacements w out of the plane of the specimen [3] are used to do this. All three displacement vector components can be determined experimentally by using the holographic interferometry method [4]. Two schemes are possible for obtaining the initial information, the hologram recording: an extra-axial [4] and opposing beam [5] scheme. The second scheme is preferable in investigations of real structures or their elements since it permits consolidating the recording medium on the surface of the object being considered, the applied holographic interferometer, which substantially reduces demands for vibration-insulation of both the testing equipment and the optical elements. Moreover, recording of the displacements of the object as a single whole is eliminated, which simplifies processing the interference patterns.

However, in addition to the advantages, such a hologram recording scheme possesses an essential disadvantage, low hologram quality. Holograms can be restored only in the laser light beam used for the recording, the interference fringe patterns are interferograms that are observed only in beams reflected from the holograms. The former circumstance results in the appearance of speckles, which makes recording of the interference fringes difficult in areas with high displacement gradients, and the latter results in the displacement w yielding the greatest contribution to the interference fringe formation. This displacement vector

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